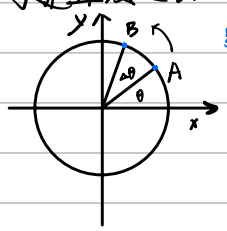
快乐学习——物理多多

圆周运动

一、匀速直线运动（匀速率，速度方向会变）



矢：$\boxed{\text{角速度 } \omega}$：$\omega=\frac{\Delta\theta}{\Delta t}$ [rad/s] $=\frac{2\pi}{T}$

注意：θ为弧度制

矢：$\boxed{\text{线速度 } v}$：$v=\frac{2\pi r}{T}=\omega\cdot r$

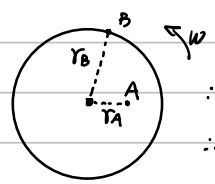
$\boxed{\text{周期 } T}$：$T=\frac{2\pi}{\omega}$

$\boxed{\text{频率 } f}$：$f=\frac{1}{T}$、$\omega=\frac{2\pi}{T}=2\pi f$

$\boxed{\text{转速 } n}$：n取r/s时 $n=f$

二、三种传动

1. 同轴传动



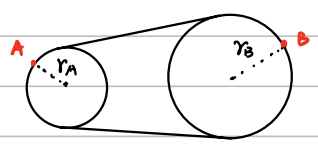
$\omega_A=\omega_B$

$\therefore v_A=\omega\cdot r_A$，$v_B=\omega\cdot r_B$

$\therefore \frac{v_A}{v_B}=\frac{r_A}{r_B}$

ω一定时，速度与半径成正比

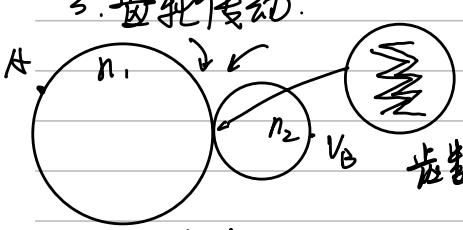
2. 皮带传动



$v_A=v_B$

$\therefore \frac{\omega_A}{\omega_B}=\frac{\frac{v_A}{r_A}}{\frac{v_B}{r_B}}=\frac{r_B}{r_A}$

3. 齿轮传动

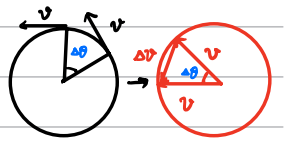


齿数：n_1、n_2

齿数：n_1、n_2

$\frac{\omega_1}{\omega_2}=\frac{n_2}{n_1}$

三、向心加速度 a_n



$a=\frac{\Delta v}{\Delta t}$，当时间t取足够短时，$\Delta v=v\cdot\Delta\theta$

$a=\frac{v\cdot\Delta\theta}{\Delta t}=v\cdot\omega=\boxed{\frac{v^2}{r}=\omega^2\cdot r}$

由于该加速与速度垂直，故不改变速度大小

只改变速度方向，称为向心加速度 a_n

$a_n=\frac{v^2}{r}=\omega^2 r$

四、向心力 F_n

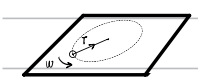
$F_n=ma_n=m\frac{v^2}{r}=\omega^2\cdot r$

※难点：以上式子很容易错误的理解为向心力 F_n 提供了向心加速度 a_n

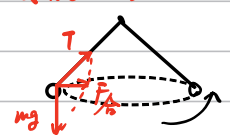
$F_合=ma$

牛顿第二定律表示为合外力提供 a。

因此，F_n 不是一个实际存在的力，而是由其他力组成的合力。



绳子拉力充当向心力。



绳子拉力与重力的合力充当向心力。

汽车水平转弯时，由地面的摩擦力提供向心力。

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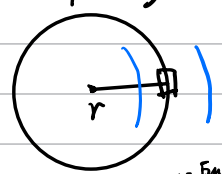
水平面内的圆周运动.

一、转弯问题.

1. 水平转弯: $F_{向}=\mu mg$

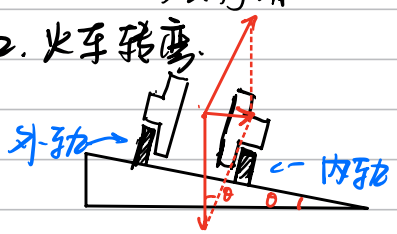


$\mu mg = m\frac{v^2}{r}$

$\mu g = \frac{v^2}{r}$

一般来说 μ, r 不变. 因此. 存在过弯的最大速度否则会打滑.

2. 火车转弯.



$F_n = mg\tan\theta = m\frac{v_0^2}{R}$

$v_0 = \sqrt{gR\tan\theta}$

当 $v=v_0$ 时. 恰好能过弯. 不挤压轨道.

当 $v<v_0$ 时 挤压内轨.

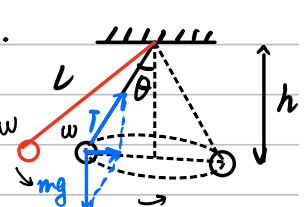
当 $v>v_0$ 时. 挤压外轨

二. 圆锥摆.

1. 单圆锥摆.

$F_n = mg\tan\theta$

$= m\omega^2 R$



(1) $\omega = \sqrt{\frac{g\tan\theta}{R}} = \sqrt{\frac{g}{h}}$ → $\omega(h)$.

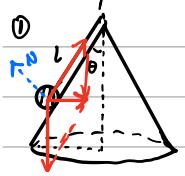
※ 相同的高度 ω 都一样.

(2) $T\sin\theta = F_n = m\omega^2 R$

$\therefore T = m\omega^2 L$

2. 临界问题

①



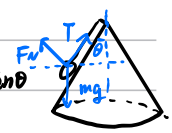
临界: $mg\tan\theta = m\omega_0^2 L\sin\theta$.

$\omega_0 = \sqrt{\frac{g}{L\cos\theta}}$

(1) $\omega > \omega_0$.

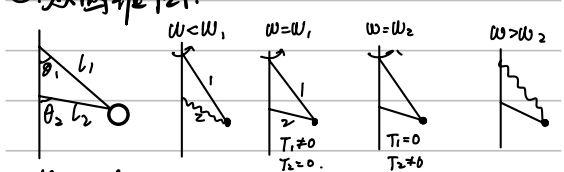
$mg\tan\alpha = m\omega^2 L\sin\alpha$.

(2) $\omega < \omega_0$.



$$\begin{cases} T\sin\theta - F_N\cos\theta = m\omega^2 L\sin\theta \\ T\cos\theta + F_N\sin\theta = mg \end{cases}$$

②. 双圆锥摆.



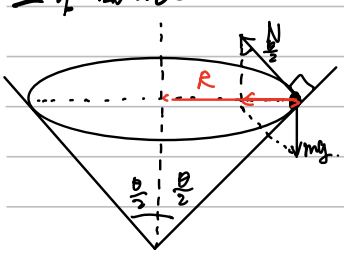
随着 $\omega\uparrow$

$m\omega_1^2 l_1\sin\theta_1 = mg\tan\theta_1$

$\omega_1 = \sqrt{\frac{g}{l_1\cos\theta_1}}$. $\omega_2 = \sqrt{\frac{g}{l_2\cos\theta_2}}$

$$\begin{cases} \omega<\omega_1 & T_2=0. \\ \omega_1<\omega<\omega_2 & T_1, T_2\neq 0 \\ \omega>\omega_2, & T_1=0. \end{cases}$$

三. 锥面滚.



$F_n = ma_n$

$mg\tan\frac{\theta}{2} = m\omega^2 R$

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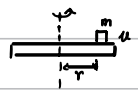
4. 圆盘 (前年高考).

① 

$f_m = \mu mg = m\omega^2 \cdot r$

$\therefore \omega_{max} = \sqrt{\frac{\mu g}{r}}$

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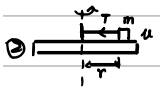
② 

$f_m = \mu mg = m\omega_1^2 r$

$T + f_m = m\omega^2 \cdot r$

$\begin{cases} \omega \le \sqrt{\frac{\mu g}{r}}, & T = 0 \\ \omega > \sqrt{\frac{\mu g}{r}}, & T \neq 0 \end{cases}$

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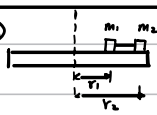
③ 

$\omega_1 = \sqrt{\frac{\mu g}{r_1}} \quad \omega_2 = \sqrt{\frac{\mu g}{r_2}}$

$r_1 < r_2 \rightarrow \omega_1 > \omega_2$

$\therefore$ 随着 ω 个. m_2 先有滑动趋势.

$\begin{cases} f_1 - T = m_1 \omega^2 r_1 \\ T + f_2 = m_2 \omega^2 r_2 \end{cases} \Rightarrow \omega.$

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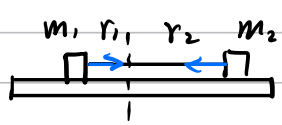
④ 

根据 $\omega = \sqrt{\frac{\mu g}{r}} \Rightarrow m_2$ 先有滑动趋势.

① $\begin{cases} T + \mu m_2 g = m_2 \omega^2 \cdot r_2 \\ T = m_1 \omega^2 \cdot r_1 \end{cases} \Rightarrow \omega.$ m_1 恰好无摩擦.

② $\begin{cases} T - \mu m_1 g = m_1 \omega^2 \cdot r_1 \\ T + \mu m_2 g = m_2 \omega^2 \cdot r_2 \end{cases} \Rightarrow \omega.$ 两木块滑动.

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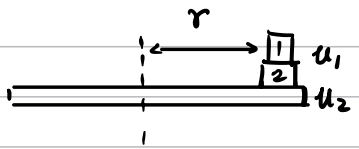
⑤ 

若 1, 2 一起动.

$\omega_A = \sqrt{\frac{u_2 g}{r}}$

若 $u_1 > u_2$ 则 $\omega_A < \omega_B$ 一起动.

若 1 动.

$\omega_B = \sqrt{\frac{u_1 g}{r}}$

若 $u_1 < u_2$ 则 $\omega_A > \omega_B$ 1 先动.

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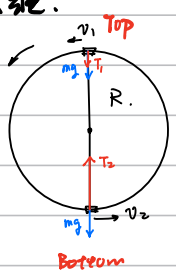
竖直面内的圆周运动

※ 竖直面内的圆周运动通常不是匀速的. 整个运动过程的受力情况难以分析. 因此通常只关注最高、最低点.

一. 绳.



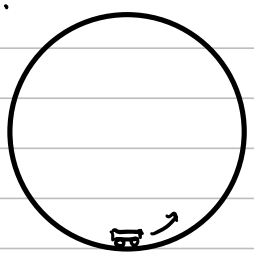
Top: $mg+T_1=m\frac{v_1^2}{R}$

Bottom: $T_2-mg=m\frac{v_2^2}{R}$

Energy: $mg2R=\frac{1}{2}mv_2^2-\frac{1}{2}mv_1^2$

单轨.

等价



临界情况:

我们关注一下 top. 由于力与拉力的合力提供向心力.

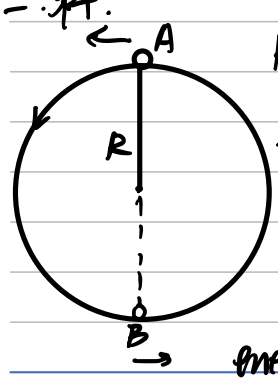
$$mg+T=m\frac{v_{top}^2}{R}$$

随着 $v_{top}\downarrow \rightarrow T\downarrow$ 当 $T=0$ 时.

$$mg=m\frac{v_{min}^2}{R} \quad \therefore v_{min}=\sqrt{gR}.$$

① 恰好能做圆周运动的最高点的最小速度.

二. 杆.



Bottom: $T-mg=m\frac{v_B^2}{R}$

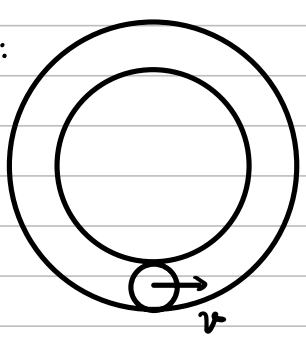
top:

$v>\sqrt{gR}$ $T+mg=m\frac{v_A^2}{R}$

$v<\sqrt{gR}$ $mg-T=m\frac{v_A^2}{R}$

energy: $mg2R=\frac{1}{2}mv_B^2-\frac{1}{2}mv_A^2$

双轨:



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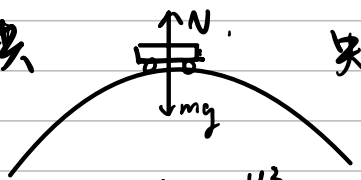
三. 拱、坑.

① 拱.

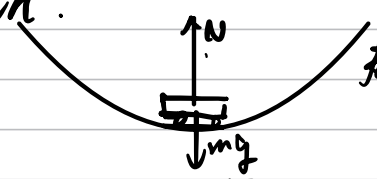


失重. → 最容易离地导致失控.

$mg - N = m\frac{v^2}{R}$

即 $mg > N$. 加速度向下.

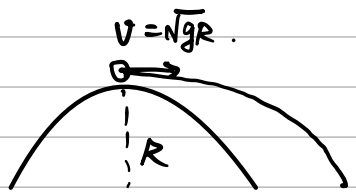
② 坑.



超重. → 最易爆胎的时候.

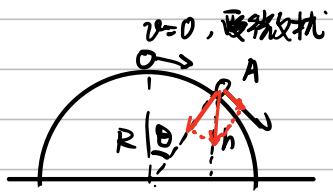
$N - mg = m\frac{v^2}{R}$, 加速度向上.

③



当速度 $v \geq \sqrt{gR}$. 小孩在最高点离开斜面做平抛运动.

④



最高点静止受微扰向右滑动.

设在 A 点脱离斜面. → $N=0$.

$$\begin{cases} mg\cos\theta = m\frac{v_A^2}{R} \\ mg(R-h) = \frac{1}{2}mv_A^2 - 0 \Rightarrow h = \frac{2}{3}R \\ \cos\theta = \frac{h}{R} \end{cases}$$

同理. 若初速度不为 0 也能求出.